Title: Spatial Sense: I Just Want to Fit In

Brief Overview:

Students need to develop spatial sense for geometric figures. This is important because the developed spatial sense leads to better understanding of more abstract concepts such as "parts of a whole" and fractions, perimeter and area relationships, and the continual growth of number sense. After completing this series of lessons they should have a working understanding of how various polygons "fit" into each other and how polygons combine to create different shapes. They should also understand how the areas of different polygons relate to each other.

NCTM Content Standard/National Science Education Standard:

Geometry
Number and Operations
Data Analysis
Measurement

Grade/Level:

Grades Four GT and Five.

Duration/Length:

This Concept Unit should require three lessons lasting fifty minutes each.

Student Outcomes:

Students will:

- Develop accurate spatial sense about geometric figures by combining pattern blocks to create multi-sided figures.
- Correctly compute the area of right triangles by comparing shapes and areas of squares and rectangles to those of triangles that "fit" into them and creating a mathematic formula to describe that comparison.

Materials and Resources:

Containers of Pattern Blocks or RS1 – Pattern Blocks Sheet
Scissors
Overhead Projector
Overhead Pattern Blocks
Overhead Markers
Colored Pencils or Crayons

Transparencies
Rulers(cm)
RS2 a and b – Student *I just Want to Fit In*
RS3 – Student *You Create It*
RS4 – Student *Fits Swell*
RS5 – Teacher *Fits Swell* Answer Key
RS6 – Student Grid Paper
RS7 a and b – Student *Make a Right At The Corner*
RS8 a and b – Teacher *Make a Right At The Corner* Answer Sheet
RS9 – Student *Connecting The Dots*
RS10 – Teacher *Connecting The Dots* Answer Sheet
RS11 – Student *Connecting The Dots* Assessment
RS12 – Teacher *Connecting The Dots* Assessment Answer Sheet
RS13 – Student *School Spirit*
RS14 – Student *Can You Make The Cut?*
RS15 – Teacher *Can You Make The Cut?* Answer Sheet

Development/Procedures:

Lesson 1

Preassessment – The students should have an introductory understanding of geometric figures and shapes and how to compute the area of rectangles and squares. These skills should have been taught in fourth grade. A review of each skill is incorporated in the lesson.

- Have students form pairs at table groups and distribute pattern block "bins" and **Resource Sheets #2 a and b, *I Just Want To Fit In,*** to each table group. Have each student get out colored pencils or crayons.
- Ask if there are any new students to our school this year or if anyone has ever entered into a new group of people before. Field responses and move the discussion into "FITTING IN". Inform students that we are going to see how geometric figures "fit in" with others and how they relate to each other in some way.
- Explain that the pattern blocks in the bin can be used to fill in the shape they see on their sheets. Have students use pattern blocks to fill in the figure in the upper left hand corner of the **Resource Sheets #2 a and b**, ***I Just Want To Fit In,*** WITHOUT OVERLAPS OR HOLES.
- Once students begin, observe different pairs to assess how successful students are at combining similar and different blocks to fill in the figure.
- Provide quiet reinforcement to those who are struggling. Instruct students to color in their figure once they have completed filling it in with pattern blocks.

Launch –

- Have students stop after about ten minutes of preassessment. Use the overhead**, Resource Sheet #2 a** (transparency), colored markers, and overhead designed pattern blocks to demonstrate one possible way to fill in the figure.
- Model how you "color in" your figure using markers and have students continue coloring in their figures.
- As they complete this, facilitate a class discussion allowing students to share their combinations verbally and on the overhead.
- Have students complete a "Gallery Walk" to observe other students combinations then return to their seats.

Teacher Facilitation –

- Ask students to share with the class how they can combine different and/or similar pattern blocks to "fit in" with other pattern blocks NOT THE FIGURE (ex: three triangles fit in with one trapezoid).
- Model on the overhead using transparent pattern blocks.
- Lead a discussion in which students articulate various combinations that "fit". Have students mimic these combinations on their desks. Model some of them on the overhead as necessary.

Student Application –

- Direct students to Table A on their **Resource Sheet #2 a** Explain how the table is set up and how the pattern blocks in your colored-in figure represent money amounts.
- Model how you count the number of different pieces used in the figure and write that number in the appropriate column.
- Model further how you multiply the number of times used column by the money amount and arrive at a "total". Ask students to assist you as you compute your figure's total value.
- Have students compute the value of their colored figure. Monitor their work and assist as necessary.

Embedded Assessment –

- After completing Table A, Challenge students to fill in "figure B" using patterns blocks that will add up to the least amount of money. They are to use Table B to record their totals.
- Repeat the same for "figure C" and Table C on Student Resource Sheet 2b using blocks that total the greatest

amount. For each figure and table students must demonstrate the use of correct pattern blocks and accurate tabulations in the tables.

Reteaching/Extension –

- For those who have understood the lesson, take them to the next step with **Teacher Resource Sheet #3, *You Create It,*** in which students are challenged to design a creature or character using pattern blocks, and then color it accurately.
- For those who have not completely understood the lesson, have them complete **Resource Sheet #4, *"Fits Swell!".*** This sheet uses the same exercise as the assessment but provides hints as to how some pieces fit into others, places values of different pattern blocks in them, and includes a partially filled in table. Answer key **Resource Sheet #5, *Fits Swell.***

Lesson 2

Preassessment/Launch- How do you find the area of a square and of a rectangle?

- Draw a square on the overhead with length of 4 cm. What is the area of the square? Draw a rectangle with a width of 6 cm and length 7 cm. What is the area of the rectangle?
- In your own words, what is area?
- Remind students that area is always written in square units. Review answers to the warm-up. The area of the square is 16 units^2 and the area of the rectangle is 42 units^2.

Teacher Facilitation-

- Review warm-up by drawing a square and a rectangle on a transparency of grid paper, **Resource Sheet #6**. Review how to find the area of a rectangle and a square.
- Pass out 2 sheets of grid paper, **Resource Sheet #6,** to each student. Students and teacher should then draw a square with a length of 6 units on each piece of grid paper. Be sure students label the length of each side and find the area of the square.
- On one of the squares draw a diagonal to form 2 congruent triangles. Cut out the square and then cut the same square along the diagonal. Allow students to fit the triangles into the other square.
- Ask students," What do you notice?" Guide students to understand that 2 right triangles create a square and that the area of 1 right triangle equals one half the area of the square.
- What is the area of 1 right triangle? What is the formula for finding the area of a right triangle? Refer back to the area of the original square. Write on the overhead the formula for

finding the area of a square. (Length x Width) or (Sides Squared)

- Introduce new vocabulary for triangles, base and height. The width of the squares is the height and the length is the base. The base can be referred to as the bottom line and the height is the vertical line. The base of the triangle is also the length of the rectangle. The height of the triangle is also the width of the rectangle. The height is perpendicular to the base. (This information is integral to the next lesson.)
- Draw a rectangle with the base of 7 units and a height of 4 units on the overhead. Students should draw the same rectangle on both sheets of grid paper.
- Repeat the process of drawing the diagonal of the rectangle and cutting one of the rectangles. Does is it have the same formula?

Student Application-

- Students will draw a rectangle with a base of 8 units and height of 3 units and another rectangle with a base of 9 units and height of 7 units.
- Students will need to determine the area of each rectangle along with the area of each triangle that creates the rectangle.
- Students will then draw two squares: one square with sides of 5 units and another square with sides of 8 units. Students should determine the area of the squares as well as the area of the right triangles that create the square.

Embedded Assessment-

- Students will complete **Resource Sheets #7 a and b *Make A Right At The Corner*.** Students will determine the area of triangles drawn on a grid, find the area when only given the dimensions, and draw triangles with a given area. Answer Key **Resource Sheets #8 a and b, *Make A Right At The Corner*.**

Reteaching/Extension-

- For extension, students will complete the bonus activity on bottom of page **Resource Sheet #7 b.**

Lesson 3

Preassessment/Launch-

- Review with students the area formula for right triangles and remediate if necessary.

- Ask students to go back into their memory banks and list the different types of triangles. What are their characteristics?
- Lead the discussion to non-right triangles. Ask student how might we find the area of non-right triangles.
- Facilitate a short class discussion that reflects on the skills learned in Lesson 2. (Using triangles within rectangles to find the base and height.)

Teacher Facilitation-

- Pass out **Resource Sheet #9, *Connecting the Dots***. Use the overhead copy of **Student Resource #9, *Connecting the Dots*** and Teacher Script **Resource Sheet #10, *Connecting the Dots*** to guide instruction.
- Describe how we will use our worksheet. We will draw what we discuss on the left and write out the steps that we take on the notes area on the right. Check for questions.
- See notes on **Resource Sheet #10.**
- Optional: Have students copy the figure from letter C of SR onto grid paper **Resource Sheet #6** and cut out the two rectangles. Students can then separate the two smaller rectangles and push them back together.
- Before moving on to the student application, remind students that the height of a triangle is always perpendicular to its base.

Student Application-

- Have students use what they have learned to find the area of the triangles on **Resource Sheet #11 *Connecting the Dots*.** Assist as needed. Answer key **Resource Sheet #12, *Connecting the Dots*.**

Embedded Assessment-

- Have students complete, **Resource Sheet #13, *School Spirit.***

Reteaching/Extension-

- For reteaching and extensions activities, have students access the website www.aaamath.com. Students will need to locate the geometry section and then locate area of a triangle. Students will have extra practice along with play games.
- Students can also access www.mathgoodies.com. Students will be allowed to print worksheets for additional help with area of squares, rectangles, and triangles.

Summative Assessment:

Students will complete the Assessment Activity, **Resource Sheet #14, *Can you Make The Cut***. Students will need to use a metric ruler to measure the dimensions of the diamond. They will apply their understanding of shapes fitting inside of shapes and computing the area of squares, rectangles, and triangles by helping create a diamond for Just In. Students will need to compute the area of the diamond. Students will then write a paragraph justifying their diamond pattern and how it "fits" Just-In's diamond specifications. Answer Key is **Resource Sheet #15 *Can you Make The Cut***.

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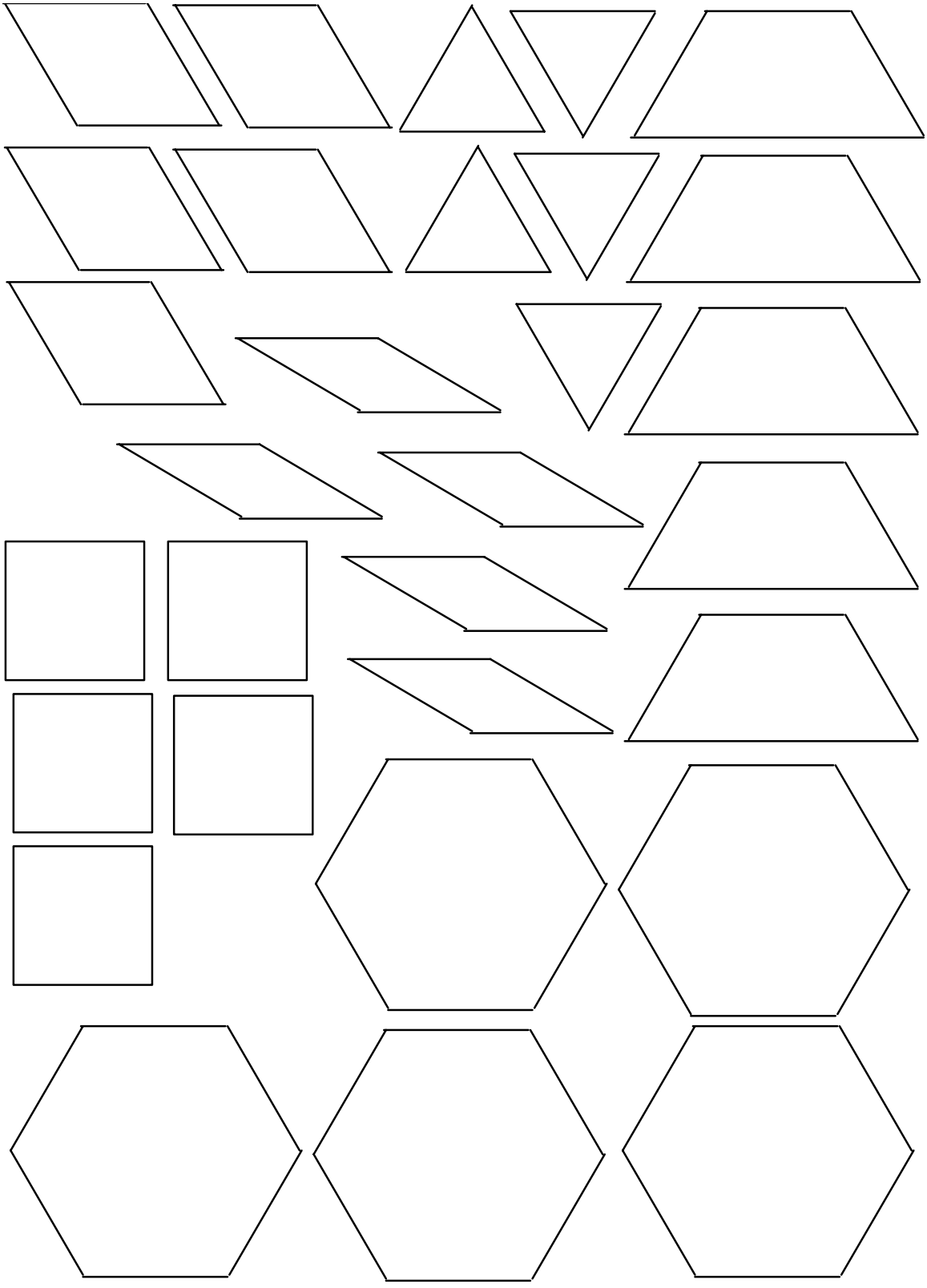
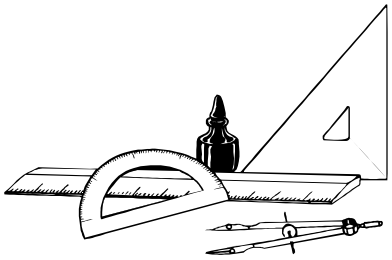
PATTERN BLOCKS





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I Just Want to Fit in!

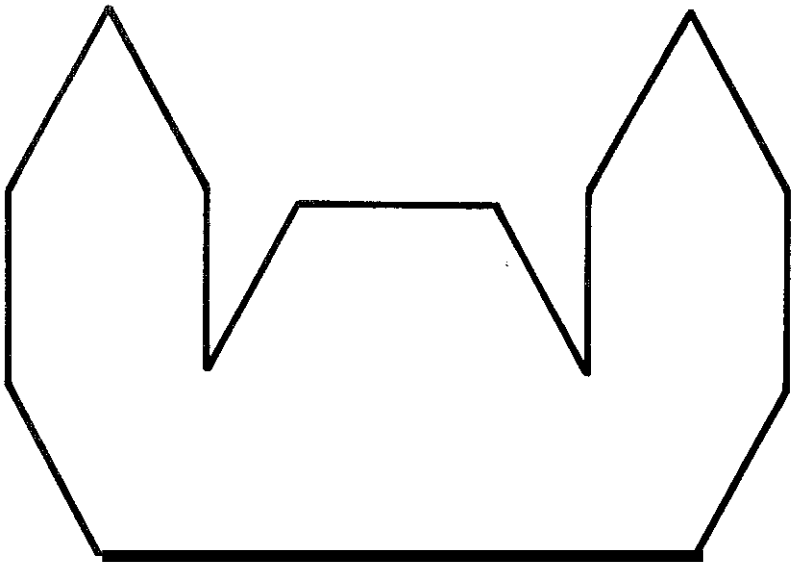
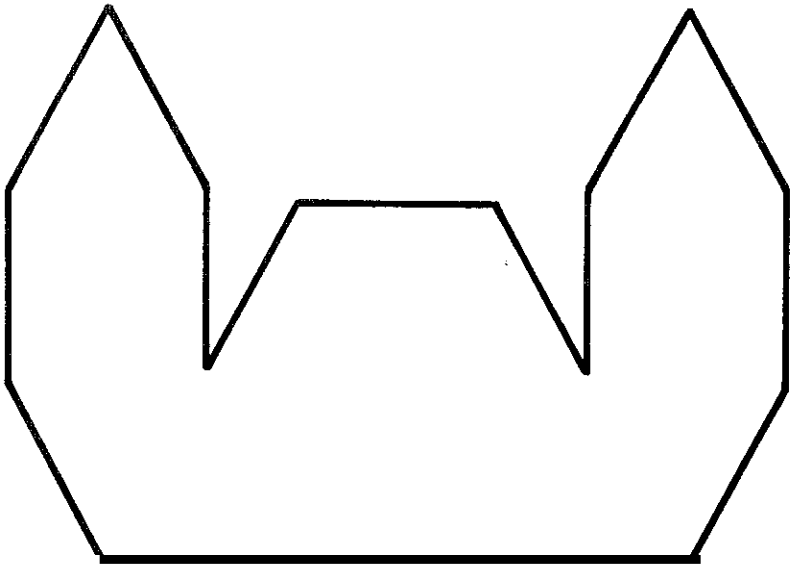


Table A			
Shape	Value	Number	Cost
▲	$0.01		
▰	$0.05		
◆	$0.10		
▼	$0.25		
■	$0.50		
⬢	$0.75		
Total			

Table B			
Shape	Value	Number	Cost
▲	$0.01		
▰	$0.05		
◆	$0.10		
▼	$0.25		
■	$0.50		
⬢	$0.75		
Total			



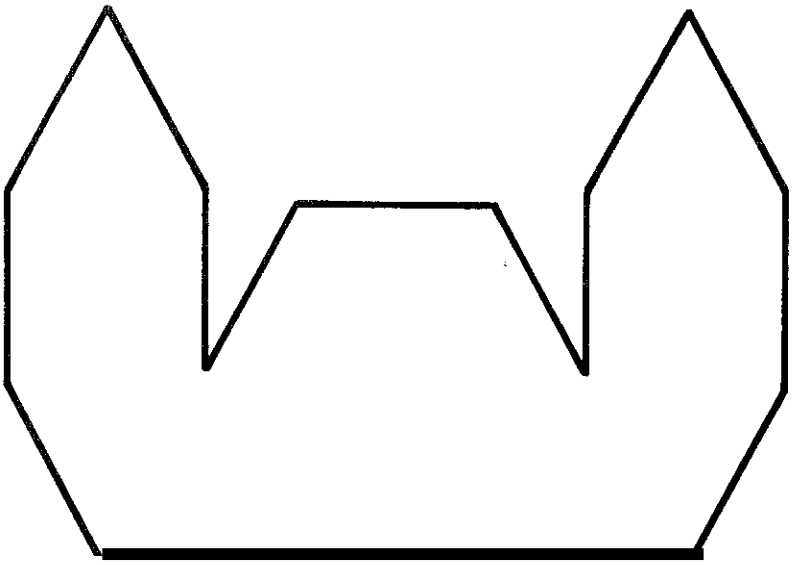


Table C			
Shape	Value	Number	Cost
▲	$0.01		
▰	$0.05		
◆	$0.10		
▼	$0.25		
■	$0.50		
⬢	$0.75		
Total			

Name _____

You Create It

Your fifth grade reading class is creating a picture story-book that will help younger grade students learn about pattern blocks and the shapes they can make. You have been selected to help create and illustrate the story using pattern blocks. First, on the back of this sheet brainstorm a real-life figure(s) you can build using pattern blocks and the words in a story that would accompany the figure(s). Second, build and color the figure in the illustration box and then write the portion of the story on the lines below.

_____

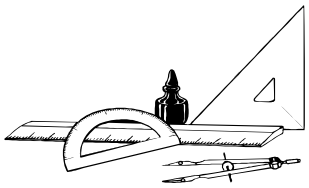
_____

_____

_____

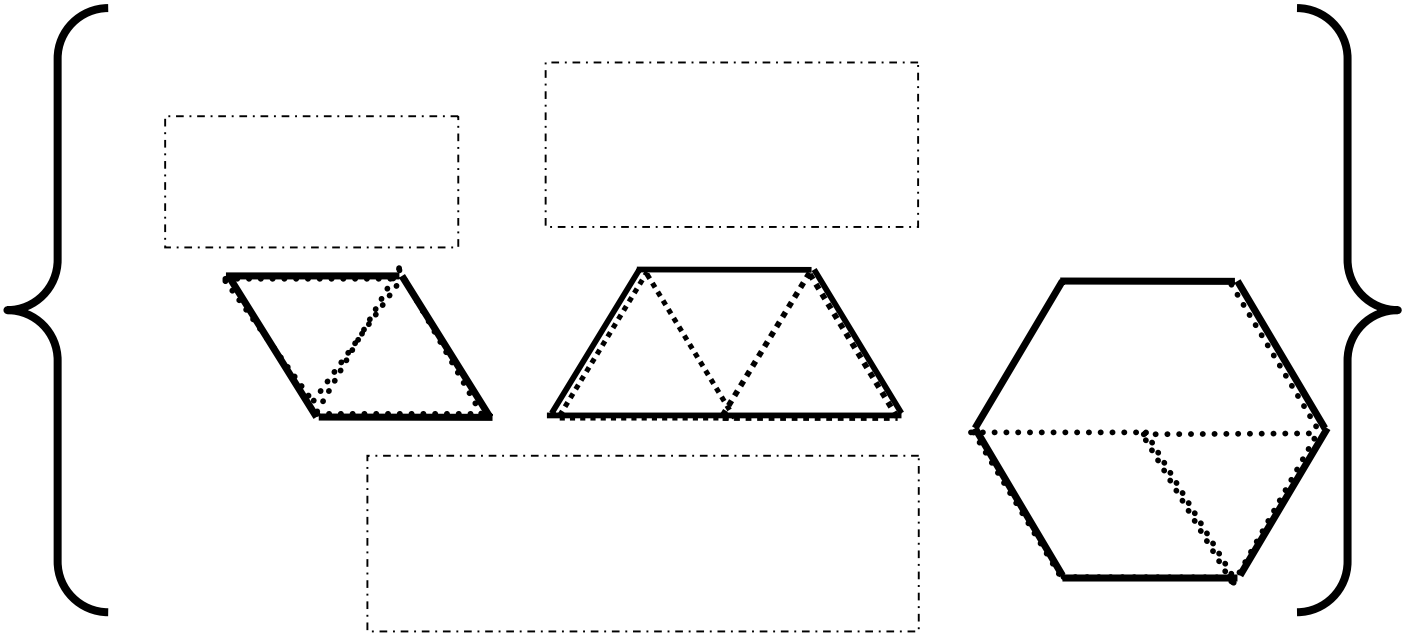
_____

Name _____



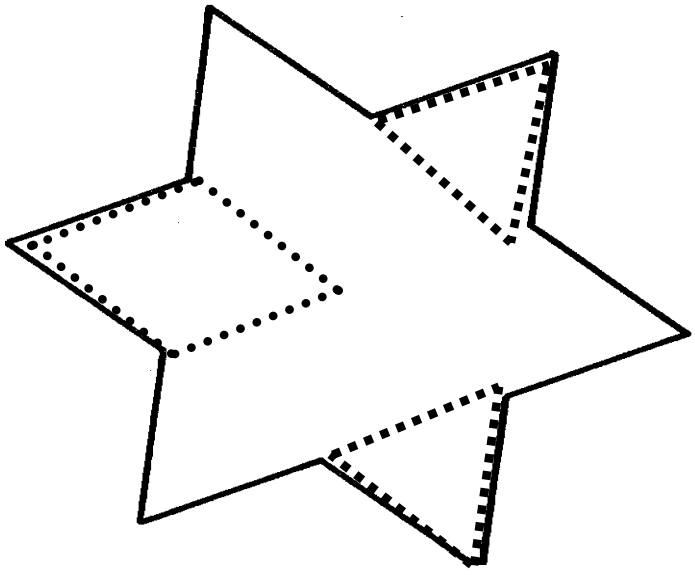
Fits Swell!

Below you will see examples of how different geometric pattern blocks (shapes) fit into larger, different pattern blocks (shapes). Below the examples is a figure that is partially filled in. Can you complete the "fill in"?



What's Missing?

There are some shapes missing from the figure to the right. Using your pattern blocks, fill in the "empty" spaces. Explain why you knew the shapes would fit.
To complete the "filling in" of the figure I used _____and_____ _____. I used them because I know that _____
_____
_____
_____
_____
_____



Name _____

Fits Swell!

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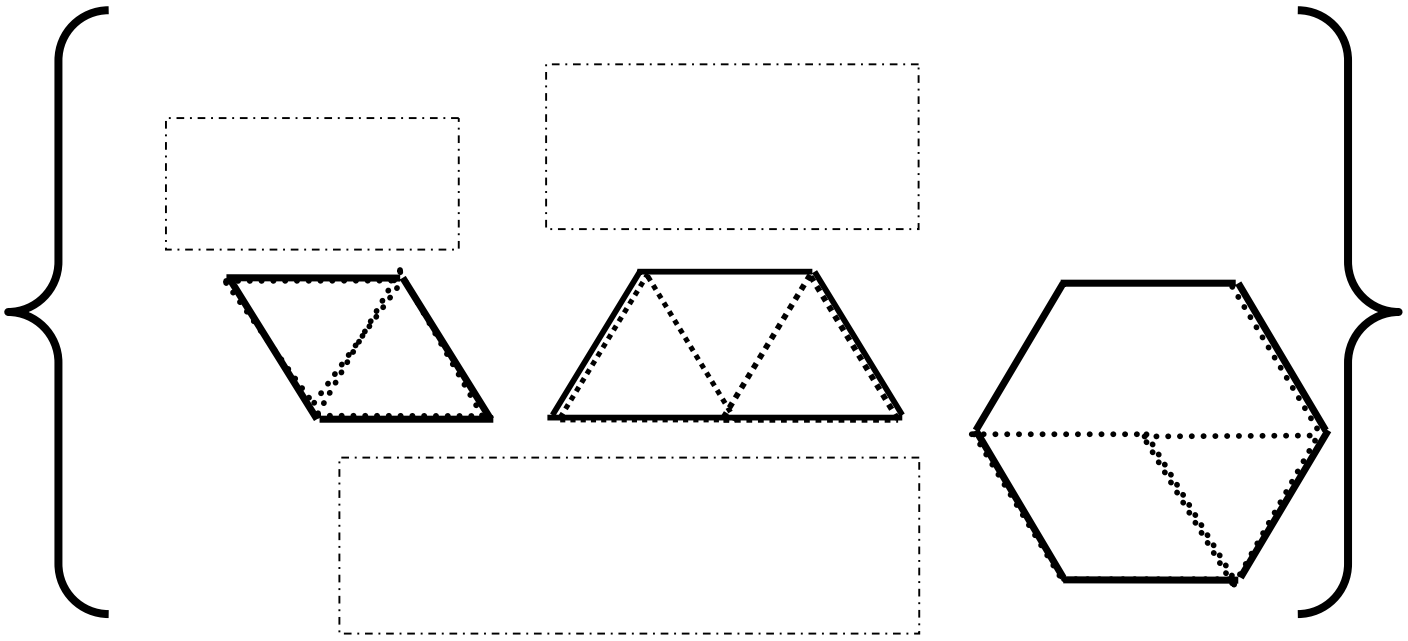
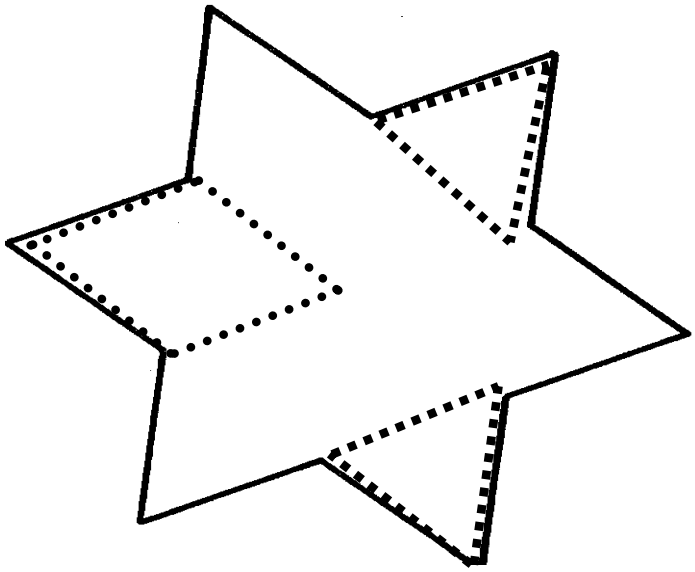


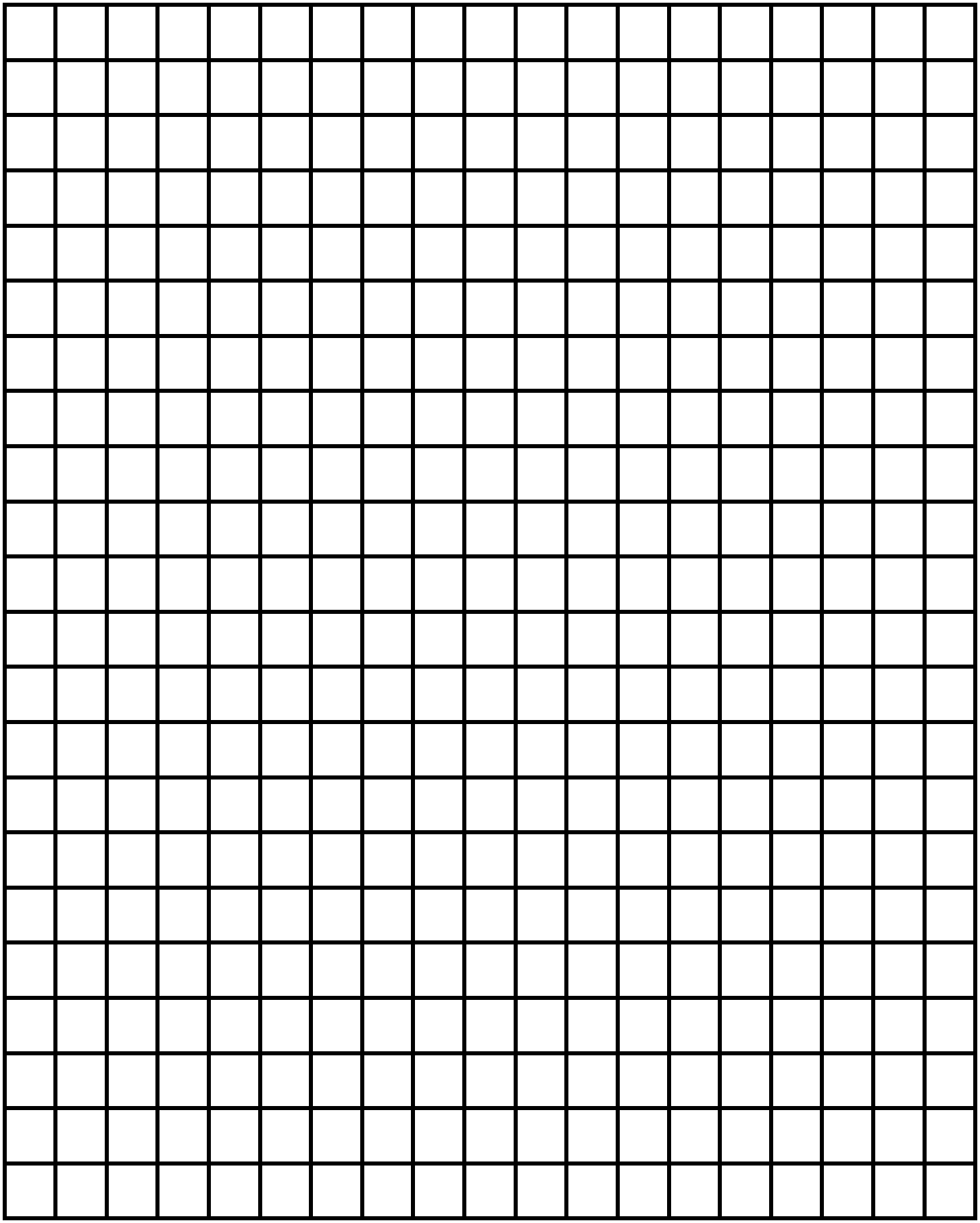
What's Missing?

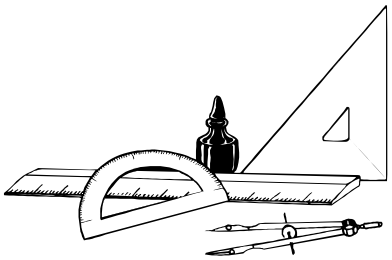
There are some shapes missing from the figure to the right. Using your pattern blocks, fill in the "empty" spaces. Explain why you knew the shapes would fit.
To complete the "filling in" of the figure I used ___**ANSWERS VARY**_____ _____. I used them because I know that _____ _____

_____**ANSWERS VARY**_____



Name _____ **Date** _____





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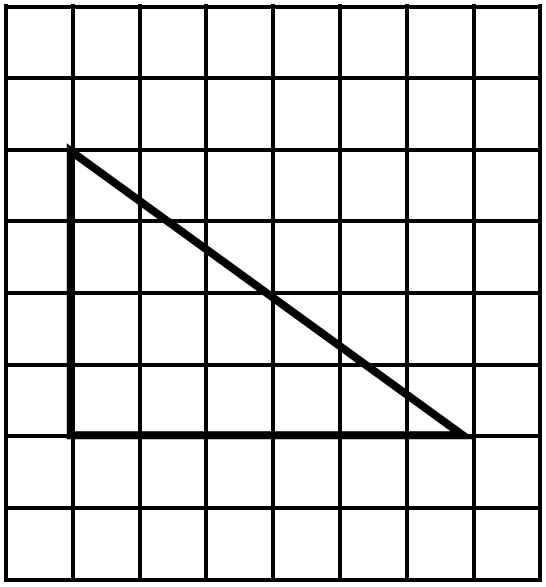
Date: _____

Take a Right at the Corner…

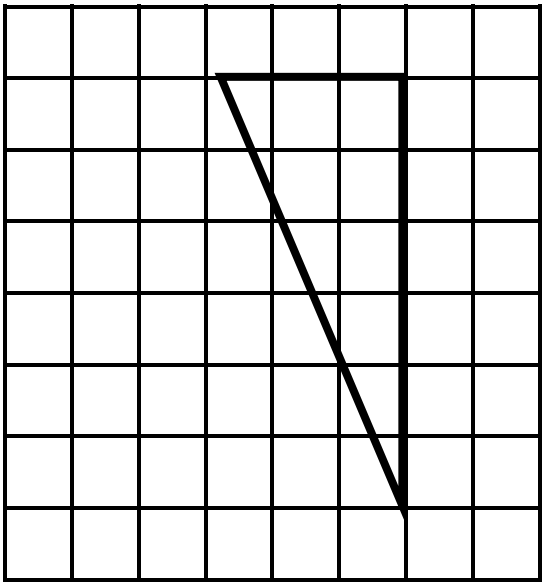
What is the area of each triangle? Label the base and the height. Remember to record your answer in square units!

1.



_____

2.



_____

3. What is the formula for the area of a triangle? Explain how you know it works.

_____

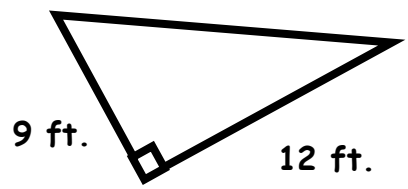
_____

_____

_____

Use the formula for area of a triangle to find each area. Write the formula, substitute the numbers, and give the answer.

4.



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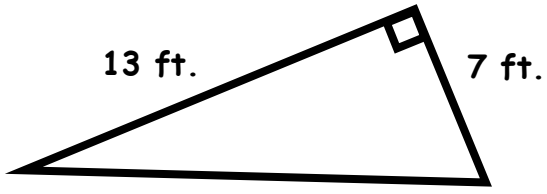
5.



_____

6. A right triangle with a base of 15 and a height of 10.

_____

7. A right triangle with a base of 18 and a height of 9.

_____

8. Base = 11
   Height = 12
   Area = _____

9. Base = 24
   Height = 6
   Area = _____

10. Base = 35
    Height = 5
    Area = _____

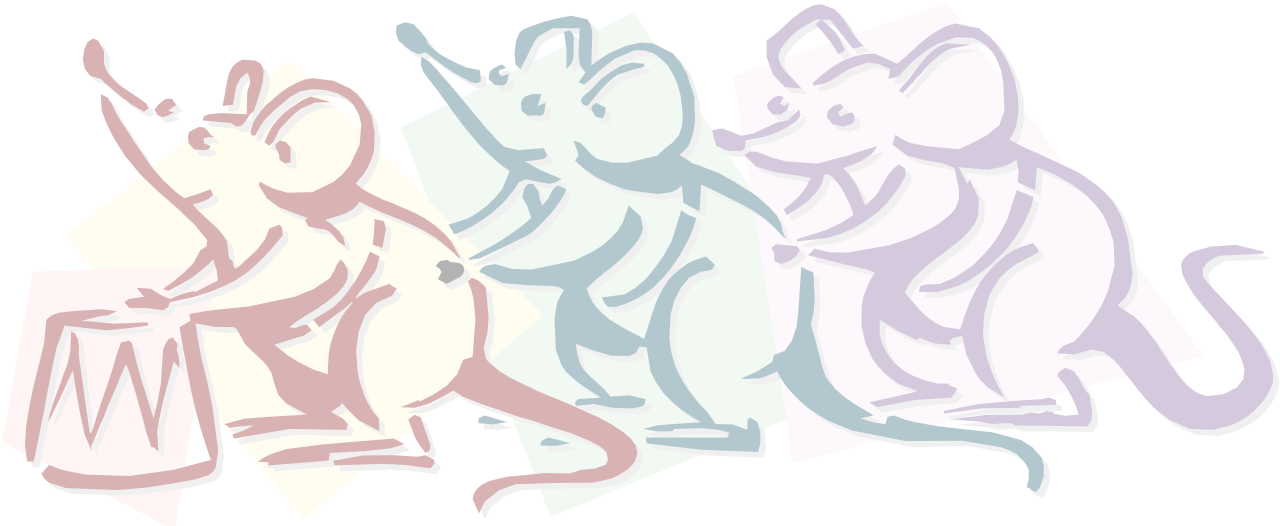
11. Base = 8
    Height = 57
    Area = _____

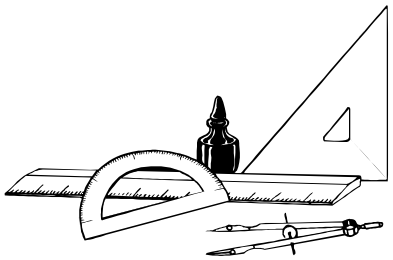
Bonus!!

Imagine you are helping to create the set for the play version of "The Three Blind Mice." Your job is to cover cardboard with paper to make the cardboard look like three different triangular pieces of cheese.

You have 12 square feet of light yellow, 20 square feet of medium yellow, and 24 square feet of dark yellow paper. The stage manager is very picky and says you may only use one color for each piece of cheese and you cannot have any paper left over.

What should the base and height of each triangular piece of cardboard be? Explain how you got your answer.





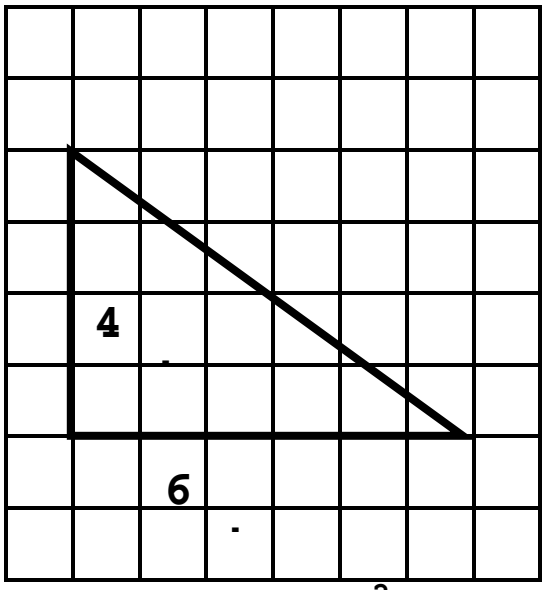
Name: _____

Date: _____

Take a Right at the Corner…

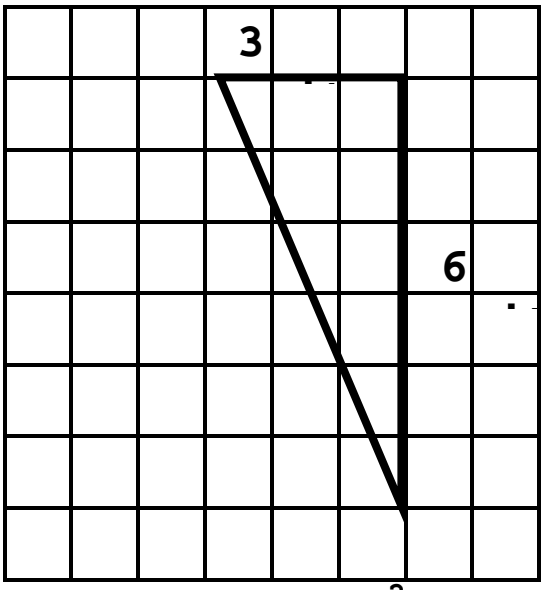
What is the area of each triangle? Label the base and the height. Remember to record your answer in square units!

1.



__12 units ^{2}___

2.



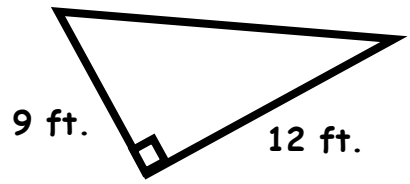
__9 units ^{2}___

3. What is the formula for the area of a triangle? Explain how you know it works.

Sample answer: The formula for the area of a triangle is ½(base x height). I know it works because the formula for a rectangle is base x height, and two congruent triangles make up a rectangle. The area of the rectangle is the same as the two triangles, so the area of one triangle is half the area of the rectangle.

Use the formula for area of a triangle to find each area. Write the formula, substitute the numbers, and give the answer.

4.

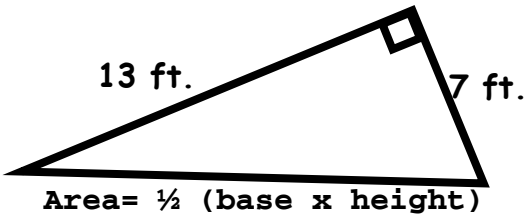


Area= ½ (base x height)

½ (12 x 9)

___54 ft ^{2}__

5.



Area= ½ (base x height)

½ (13 x 7)

___45.5 ft ^{2}__

6. A right triangle with a base of 15 and a height of 10?

Area= ½ (base x height)
½ (15 x 10)
75 units ^{2}

7. A right triangle with a base of 18 and a height of 9?

Area= ½ (base x height)
½ (18 x 9)
81 units ^{2}

8. Base = 11
Height = 12
Area = ½ (base x height)
½ (11 x 12)
66 units ^{2}

9. Base = 24
Height = 6
Area = ½ (base x height)
½ (24 x 6)
72 units ^{2}

10. Base = 35
Height = 5
Area = ½ (base x height)
½ (35 x 5)
87.5 units ^{2}

11. Base = 8
Height = 57
Area = ½ (base x height)
½ (8 x 57)
228 units ^{2}

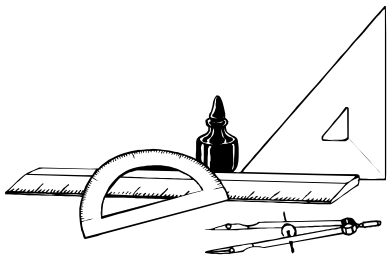
Bonus!!

Imagine you are helping to create the set for the play version of "The Three Blind Mice." Your job is to cover cardboard with paper to make the cardboard look like three different triangular pieces of cheese.

You have 12 square feet of light yellow, 20 square feet of medium yellow, and 24 square feet of dark yellow paper. The stage manager is very picky and says you may only use one color for each piece of cheese and you cannot have any paper left over.

What should the base and height of each triangular piece of cardboard be? Explain how you got your answer.

Answers will vary.



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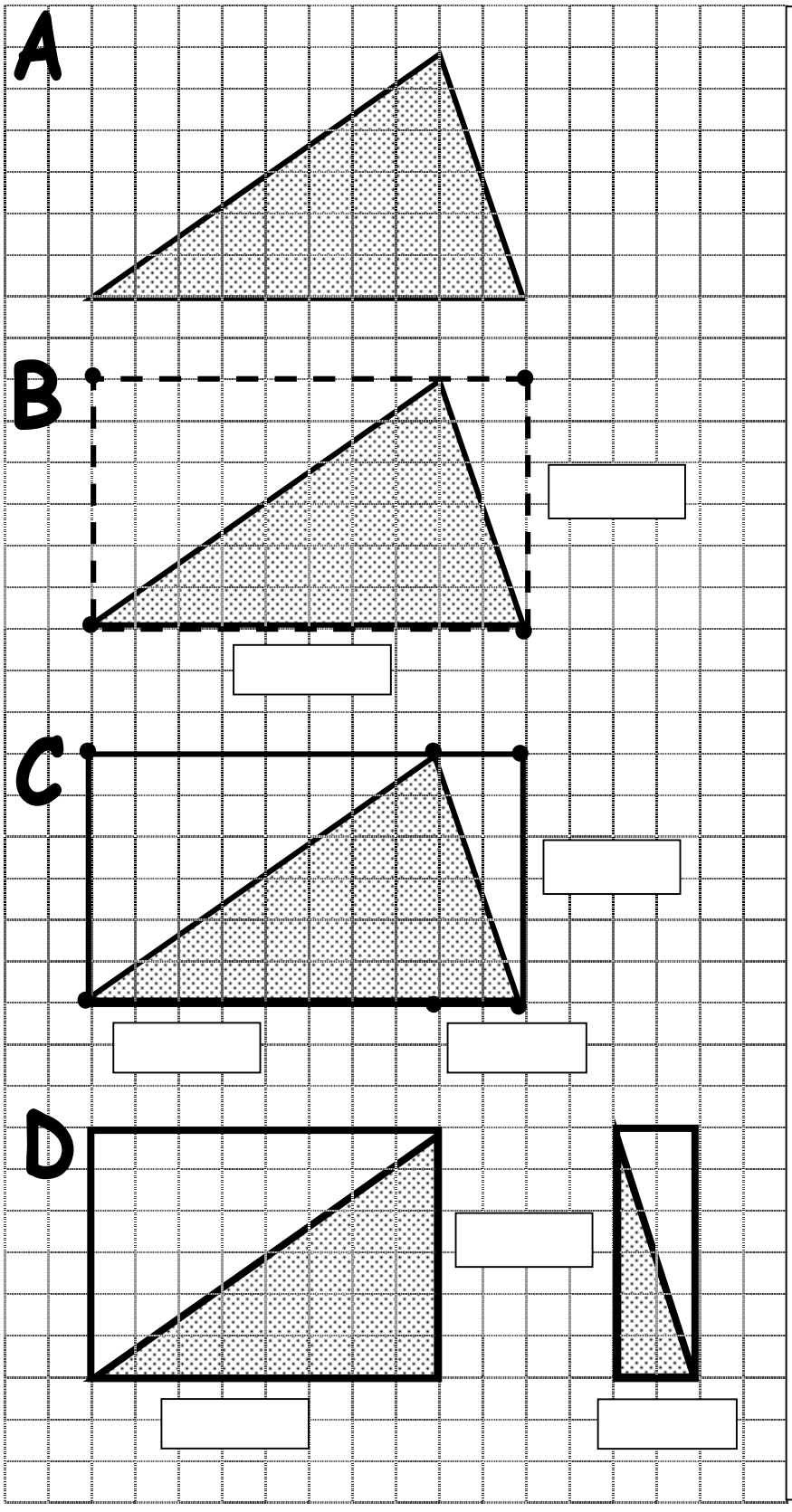
Connecting the Dots



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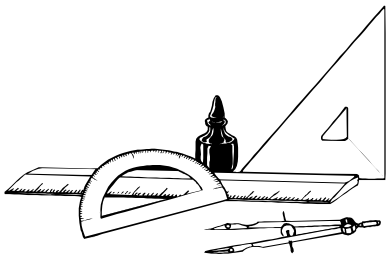

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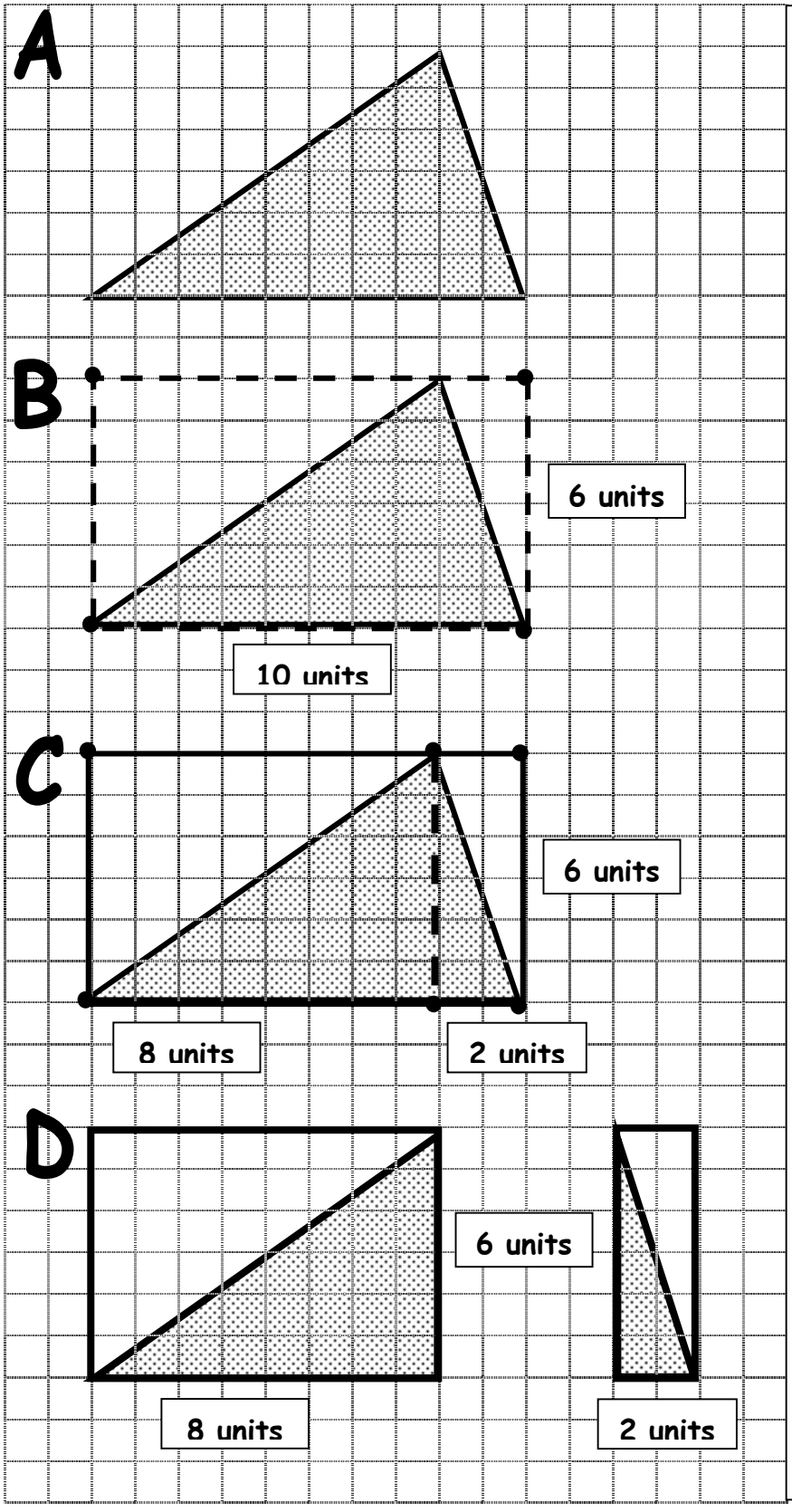
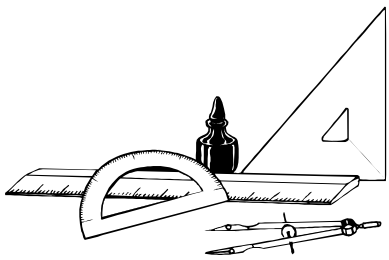
Connecting the Dots



- We need to find the area of triangle A.
- How do we find the area of a right triangle? *(½ base times height)*
- Why does it work? *(To get the area of a rectangle, you have to take the base and multiply it by the height. The area of a right triangle is ½ of the rectangle around it.)*
- Let's see if it works with other kinds of triangles. Connect the dots around the triangle next to letter B to create a rectangle. Fill in the boxes with the labels. What are the dimensions of the rectangle? *(The base is 10 units and the height is 6 units.)* What is the area of the rectangle? *(60 units squared.)* Does this help us? *(Variety of answers and discussion. Someone may come up with the next step- dividing the rectangle into two rectangles.)*
- Take your pencil and put a dot on the very tip of the triangle at letter C. Trace a line down so it makes a perpendicular line that connects with the base of the triangle. What do you notice? *(The big rectangle is now 2 smaller ones and the big triangle is also 2 smaller ones.)* Write labels in the boxes.
- Write labels in the boxes at letter D. What are the areas of the 2 rectangles? *(48 units squared and 12 units squared)* What are the areas of the two triangles? (24 units squared and 6 units squared) What is the area of the rectangle at letter B?
- What do you notice? What connections can you make? * If the students do not come up with the following statements on their own, make sure they understand them.*
  - *If you push the two figures together, you get the figure at letter C.*
  - *The triangles are the same as the ones in letter C.*
  - *If you add the areas of the rectangles in letter D, you get the same area as the rectangle at letter B.* **Why?** *When you push them together, they have the same dimensions.*
  - *If you add the areas of the triangles in letter D, you get the area of the triangle in letter A.* **How do you know?** *The 2 bases of the rectangles in letter D add up to 10, which is the same as the base of the triangle in letter A and both of the heights are the same.*
- Does ½ base times height work on a triangle that isn't a right triangle? The base of the triangle is 10 and the height of the triangle is 6. That gives us an area of 30. The rectangle around the triangle has an area of 60. What was the area of the two triangles in letter D, combined? *The area is 30. It works!*



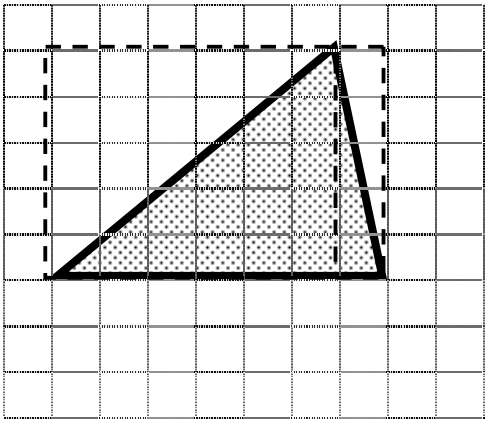
Name: _____

Date: _____

Connecting the Dots

Find the area of each triangle.

1.



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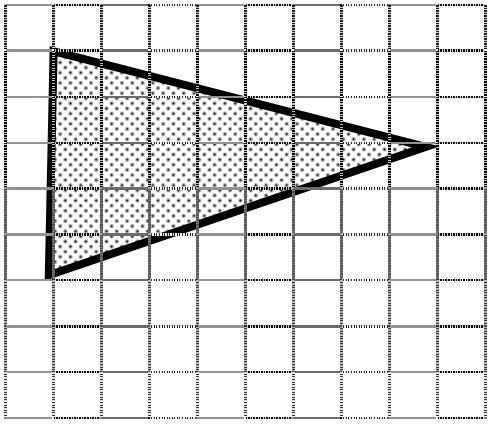
2.



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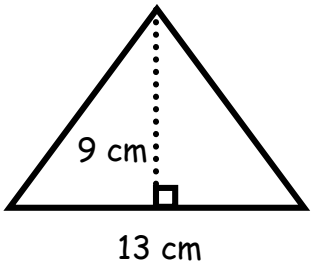
3.



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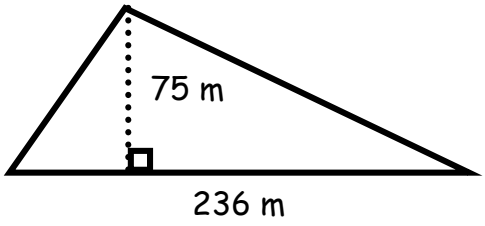
4.



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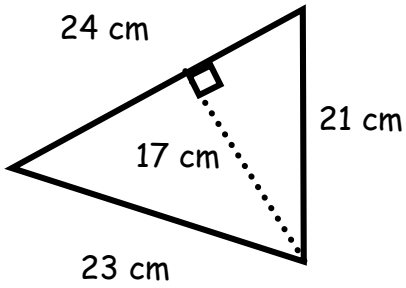
5.



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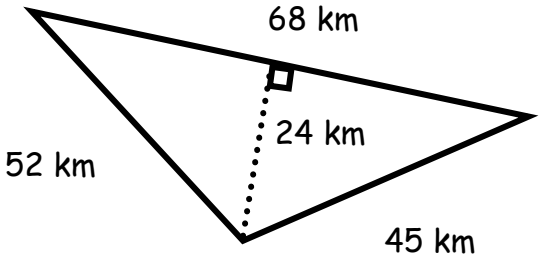
6.



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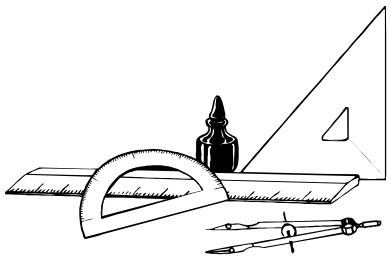


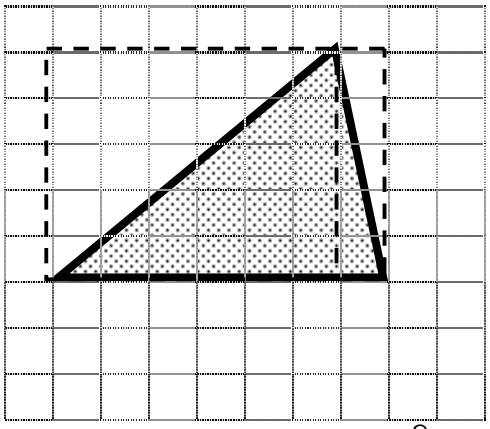
Name: _____

Date: _____

Connecting the Dots

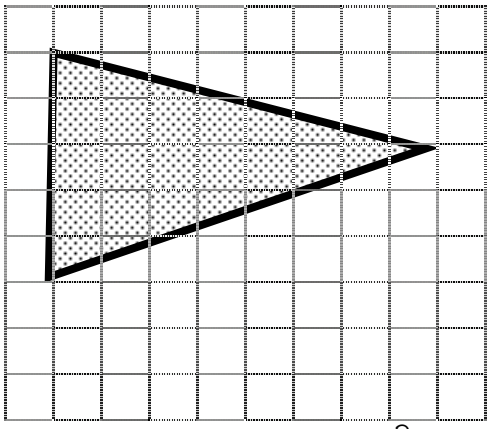
Find the area of each triangle.

1.



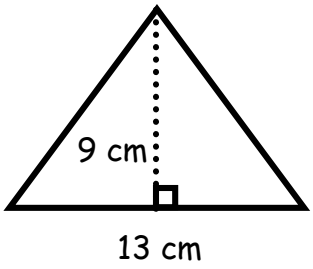
__17.5 units ^{2}__

2.



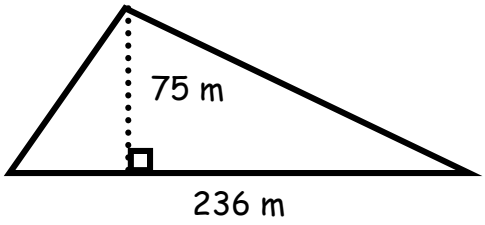
___20 units ^{2}__

3.



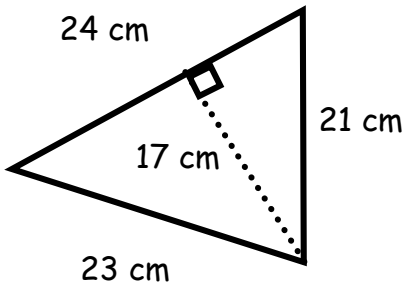
___58.5 cm ^{2}___

4.



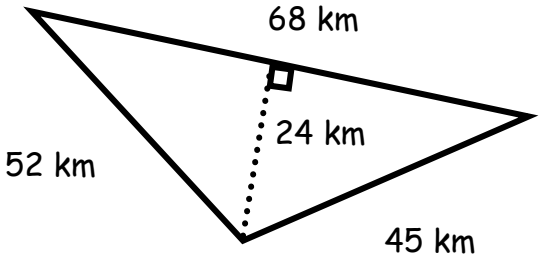
__8,850 m²___

5.



__204 cm ^{2}____

6.



__816 km ^{2}____

School Spirit

Design 3 different pennants of different areas to be sold at the school store. Each pennant must be an isosceles triangle, 2 sides are equal. The cost of the material to create the pennants is as follows:

$0.25 a square unit for felt

$1.20 a square unit for construction paper

$0.74 a square unit for foil

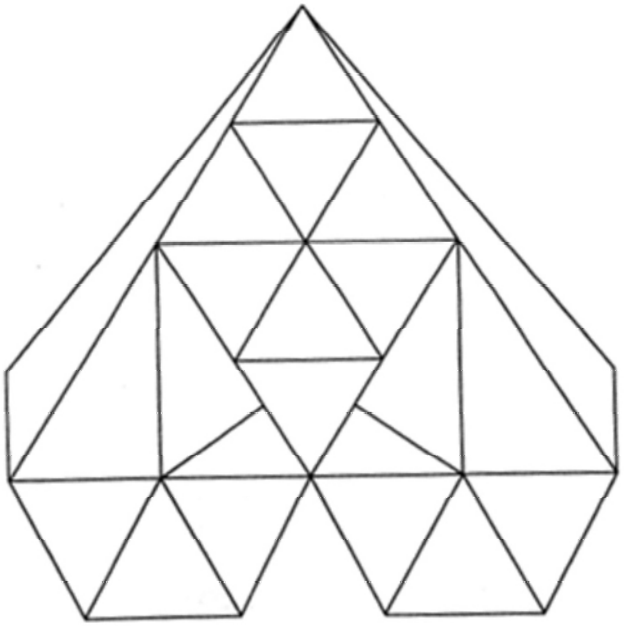
Each pennant must be created from different materials. After finding the area of each pennant, determine the cost for creating each pennant. Remember each pennant needs to have a different area, needs to be an isosceles triangle, and be created from a different material.

Choose one of your pennant designs. Explain how the area of the pennant was determined and how the cost was determined. Be sure to use math vocabulary in your explanation.

Name _____

Can You Make the Cut?

You have just returned to your shop "Diamonds R' Us" from the Smithsonian Institute where you have been honored for making the best cuts on a diamond by any jeweler. They have even put the diamond on display. Each of your specially-made blades can cut 50 cm^2 of diamond before becoming dull. No one but you knows how to design the blades you use.



A knock comes on the door of your shop and in walks "Just-in" the former star in popular band "Sink-N". He explains to you that "Bitny", his long lost girlfriend, has agreed to get back with him if he can provide her with the most beautiful diamond in the world. He shows you the rough diamond he has purchased (Wow, it's huge!). You take a huge gulp because you remember that you have only one special blade left and "Just-in" needs this diamond completed by tomorrow. In the center of this paragraph is a two dimensional sketch of the diamond that "Just-in" wants to give "Bitny". Measure each triangle with a metric ruler (round to the nearest cm) and find the total area of all the triangles that need to be cut.

Show your work on the reverse side of this sheet.

Will your blade last for all the cuts? _____ How did you determine your answer? _____

_____

_____

If you were paid $2,300 for each triangle cut or $1,200 for every cm^2 you cut, which payment option would you choose? _____ Explain _____

_____

_____

"Just-In" calls you just after you have measured all the "cut areas" and asked you if you can make fewer cuts on the diamond. Are there different shapes that can be cut that still have the borders of combined triangles? If so what is the smallest figure? The largest? Outline three new cut shapes with colored pencils or crayons. _____

_____

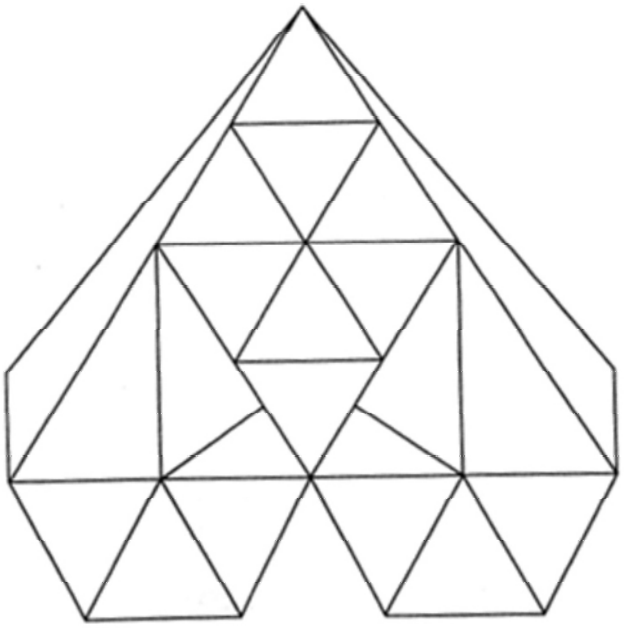
_____

_____

Name _____

Can You Make the Cut?

You have just returned to your shop "Diamonds R' Us" from the Smithsonian Institute where you have been honored for making the best cuts on a diamond ever by any jeweler. They have even put the diamond on display. Each of your specially-made blades can cut 50 cm^2 of diamond before becoming dull. No one but you knows how to design the blades you use.



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Show your work on the reverse side of this sheet.

Will your blade last for all the cuts? **_yes__** How did you determine your answer? **_Total area is 44 cm. I measured the base and height of each triangle and added them together. I didn't have to measure the area of congruent triangles, I just added them together._____**

_____

_____

If you were paid $2,300 for each individual triangle cut or $1,200 for every cm^2 you cut, which payment option would you choose? **I would choose the $1,200 for each cm^2**_____

Explain **_$2,300 x 22 total triangles = $50,600 and $1,200 x 44 cm^2 = $52,800___**

"Just-In" calls you just after you have measured all the "cut areas" and asked you if you can make fewer cuts on the diamond. Are there different shapes that can be cut that still have the borders of combined triangles? If so what is the smallest figure? The largest? Outline three new cut shapes with colored pencils or crayons. **_Answers vary – Smallest figure is the parallelogram. Largest is large center triangle._____**